Mathematical analysis of corneal remodelling after intracorneal ring surgery in keratoconus

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Abstract

\* AIM: To represent mathematically the intersection between the ectatic corneal geometry and the plane of intracorneal ring implants (ICRS) in order to determine the corneal response to ICRS surgery in keratoconus (KC). Thereafter, to present the concept and early results of a newly derived topography-guided nomogram for ICRS surgery for the treatment of keratectasia.

\* METHODS: The corneal rings plane intersection was modelled to a conic section. Ring effect was the result of: the ring size, position (steep vs flat), location (distance from the geometric centre of the cornea), and the discrepancy between the ring’s curvature and the tunnel’s curvature. Femtosecond laser was used to create the tunnels and the incision sites were chosen according to the nomogram in order to place the thickest ring in the steepest portion of the cornea regardless of the astigmatism axis of refraction.

\* RESULTS: The conic section had a more prolate shape in the steep area of the cornea than in the flat area, depending on the corneal sagittal curvature. Equal ring size had more flattening effect in the steep area than in the flat area. Thick segment should be implanted under the steep portion of the cornea regardless of the cylinder axis of refraction. Single segment in the steep area was sufficient in early and moderate cases of KC. The new nomogram provided more topographic regularity with significant reduction of astigmatism and better improvement in uncorrected visual acuity (UCVA) and best-corrected visual acuity (BCVA) than the conventional nomogram.

\* CONCLUSION: The newly derived nomogram can produce better results than the conventional nomogram. Moreover, based on this concept, a new nomogram can be integrated into the femtosecond laser software to create topography-guided, customized, elliptical tunnels with modified focal asphericity that allows for customized focal flattening of the irregularly steepened ectatic cornea.

\* KEYWORDS: keratoconus; intracorneal ring; mathematical model; cornea; topography guided; nomogram

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INTRODUCTION

Keratoconus is a bilateral non-inflammatory degenerative disease of the cornea. It is characterized by progressive thinning and steepening of the corneal apex, which achieves a conical shape in advanced stages. The resultant irregular astigmatism and myopia induce significant visual complaints. The insertion of intrastromal corneal ring segments (ICRS) is one of the treatment options that have been gaining popularity during the last decade. First evaluated as a treatment for mild to moderate myopia, these polymethyl methacrylate (PMMA) segments modify the corneal curvature by an arc-shortening, flattening effect and have proven especially effective in reducing the irregular astigmatism caused by keratoconus[1]. Prior to implantation, a tunnel is created at a 70%-80% corneal depth, either by manual dissection, or with the femtosecond technology, the latter having a better safety-accuracy profile[2-5]. Once inserted within the tunnel, the ICRS interacts with the corneal tissue and produces local flattening of the cornea. This flattening effect is believed to be related to both the geometrical properties of the ICRS (shape, arc length and thickness), and to its position relative to the corneal steep axis[6]. Nomograms are proposed by the rings’ manufacturer to help physicians in choosing the appropriate segment that would induce the desirable flattening effect. However, a lot of controversy remains concerning many technical issues like the incision site-whether it should be performed over the steep vs the flat axis or not and like the number of rings to use and whether to insert superior rings or not[7-13]. Moreover, although the surgical aspect of ICRS implantation is well mastered and the overall results are good, nomograms still can’t predict the exact flattening power of the ring once it is inserted in the corneal stroma and unexpected variations are not exceptional.
We believe that this variation is due to the poorly understood biomechanical interactions between the ring and the stromal tunnel, element that is not addressed in conventional ring nomograms. In this article, we try to mathematically explain the ring-cornea biomechanical interaction with a resultant nomogram that might be useful clinically in order to achieve better results from ICRS surgeries in keratoconic corneas.

**MATERIALS AND METHODS**

During applanation while creating a femtosecond-assisted channel for ICRS implantation, the corneal surface becomes completely flat while performing the side cut. Once the applanation cone is removed, the cornea returns to its original curved appearance. This means that, particularly in eyes with keratectasia, the steep meridian will be short while the flat meridian will be long, which can cause the formation of oval-shaped tunnels. Our focus in this study was on the ring-corneal tissue interaction happening at the level of the corneal oval tunnel. Biomechanical effects of the ring, which can be exerted as hoop stress or tangential stress, and mass effect are mediated through the contact between the ring and the corneal tissue within the tunnel itself. In order to understand this effect, we tried to mathematically model the shape of the tunnel created during corneal dissection in normal vs ectatic corneas and analyze the ring-corneal tissue interaction according to the tunnel shape and dimension and ring position inside the tunnel. Analysis of different clinical scenarios was performed and clinical application to this analysis was implemented.

**RESULTS**

Normal corneal shape can be modulated to a dome shape, which is symmetrical around the vertical axis, while the ectatic corneal shape can be modulated to an oblique circular cone (Figures 1 and 2). An oblique circular cone can be considered as an elliptical cone. In ICRS surgery, we use the femtosecond laser for the creation of the intrastromal tunnel which applanates the cornea at its perpendicular axis, and then creates a cut that is perpendicular to the corneal vertical axis (z axis). The tunnel shape is modulated as the result of the intersection between a plane cutting the cornea perpendicular to its vertical axis and the conic shape. Accordingly to the rules of conic section, the tunnel shape in normal cornea is a perfectly symmetrical circular shape [14] (Figure 1). The intersection of an oblique cone with any plane is a conic section (C).

![Figure 1 The rules of conic section](image)

**Figure 1 The rules of conic section** A: The conic sections are the non-degenerate curves generated by the intersections of a plane with one or two nappes of a cone. The opening angle of a right cone is the vertex. For a plane perpendicular to the axis of the cone, a circle is produced. For a plane that is not perpendicular to the axis and that intersects only a single nappe, the curve produced is an ellipse or a parabola. The curve produced by a plane intersecting both nappes is a hyperbola. B: Dandelin sphere relates many properties of the conics to the cone and the curve generated by the intersections of a plane perpendicular to the vertical axis of the sphere is a circle.

![Figure 2 Intersection of an oblique cone with any plane](image)

**Figure 2 Intersection of an oblique cone with any plane** An ectatic cornea (A) resembles an oblique cone (B); an oblique cone can be considered an elliptical cone. The intersection of an oblique circular cone with any plane is a conic section (C).

During ICRS surgery, the cornea is aplanatic and dissected perpendicularly to its vertical axis (same apply to the mechanical dissection). Hence, the variable “z” in equation (1) would be a constant and the whole equation of the curve will be in equation (2):

\[
x^2/a^2 + y^2/b^2 = z^2/c^2
\]

(1)

\[
x^2/a^2 + y^2/b^2 = constant
\]

(2)

Equation (2) is the equation of an ellipse and not a circle. That would conclude that tunnel shape in ectatic cornea is elliptical and not circular. This elliptical shape follows the rules of elliptical conic section: the closer to the apex, the
steeper the elliptical section. Also, the smaller is the vertex angle “opening angle”, the steeper the elliptical section. This is a mere approximation of the corneal geometry in keratoconus. In advanced keratectasia, the corneal geometry is severely distorted and one half of the cornea is much steeper than the opposite one in the same meridian. According to the previous analysis, we conclude that the tunnel shape created in ectatic cornea during ICRS surgery is not a circular shape but an asymmetrical elliptical shape which resembles the combination of 2 halves of 2 elliptical curves, the flat side corresponding to the flat-superior cornea and the steep side corresponding to the steep-inferior part of the cornea (Figure 3). Once the ring is inserted in a tunnel, apart from the “mass effect” of the ring (which is dependent on the ring thickness), ICRS-corneal tissue interaction is modulated by the discrepancy between the ring’s curvature and the channel’s curvature. If the ICRS is inserted in a tunnel, which is steeper than the ring curve, centrifugal “hoop stress” will be generated to stretch the cornea toward the periphery and further flattening of the cornea will result. On the other hand, if the ICRS is steeper than the tunnel, centripetal “hoop stress” will be generated which may counteract the flattening “mass effect” of the ring and ring’s effect might be reversed toward steepening instead of flattening (Figure 4).

If the ring is inserted in an equally steep tunnel, only “mass effect” will be exerted. These concepts helped us design a new topography-guided nomogram for femto-laser assisted ICRS surgery. The novelty of this nomogram is that it takes into account not only the “mass effect” of the ring, but also the “hoop stress” that would be generated depending on the tunnel shape. The following are general principles illustrated by actual cases that were treated using this nomogram. The sagittal axis of corneal topography is used to percept the spatial distribution of the corneal curvatures in ectatic corneas and to determine the steepest axis (Figure 5).

The thick segment should be implanted under the steep portion of the cornea regardless of the refractive cylinder axis (Figures 6 and 7). The patient whose topography is shown in Figure 6 is a 25-year-old who had an uncorrected visual acuity (UCVA) of 20/400 in his right eye with best-corrected visual acuity (BCVA) of 20/80 with a refraction of -8.75 D (-6.75 D, axis 20). Two thick (450 μm) symmetrical segments were inserted under the steep superior and inferior areas (incision axis at around 10 degrees). Postoperatively, UCVA reached 20/50 at one month. The patient whose topography is shown in Figure 7 also improved. His BCVA was 20/50 with -16.00 D (+5.00 D, axis 70) and BCVA of 20/400. Two asymmetrical rings insertion were performed in order to enhance UCVA and BCVA. At one month postoperatively, UCVA was 20/40.

Tunnel size is adjusted according to the severity of keratoconus. In order to obtain more flattening effect, the inner and outer diameter of the tunnel can be reduced slightly (e.g. 5.9 mm inner and 6.9 mm outer diameter of the tunnel instead of 6.0 mm inner and 7.0 mm outer diameter). This slight reduction of the tunnel sizes is unlikely to produce major difficulties regarding the ring insertion. Also, the position of the tunnel can be slightly manipulated; slight shift of the tunnel toward refractive cylinder axis and the thin segment should be implanted in the opposite direction (Figure 9). Patient is 14 years old with preoperative refraction of -9.00 D (+5.00 D, axis 70) and BCVA of 20/400. Two asymmetrical rings insertion were performed in order to enhance UCVA and BCVA. At one month postoperatively, UCVA was 20/40.
the apex may result in steeper tunnel elliptical cut under the steep area of the ectatic cornea and may lead to more flattening effect of the ring that is positioned under the steep area. This maneuvering might be applied only in case of one ring insertion to reduce irregularities and without interfering with the optical axis of the cornea.

Ring’s thickness is chosen according to severity of keratoconus. Incision sites are chosen in order to place the thickest ring in the steepest portion of the cornea regardless of the axis of astigmatism.

A single large arch length ring inserted under the steep segment of the cornea in order to embrace the irregularities can address large irregularities. (e.g. 210 degrees) (Figure 10).

**DISCUSSION**

ICRS effect on keratoconic corneas is the resultant of multiple parameters. Different shapes, thicknesses (0.15 to 0.45 mm), and arc lengths (90 to 210 degrees) account for different “mass effect” inside a tunnel\(^\text{[15-16]}\). Thicker rings\(^\text{[17]}\) and higher arc lengths typically produce a higher “mass effect”. The ring’s arch length is mostly determined by the extent of the area of...
the steep cornea to be flattened. The best example is the use of the 210-degree arc length ICRS in eyes with pellucid marginal degeneration. This technique was also reported by Kubaloglu et al \cite{15} with significant flattening and improvement in UCVA. If the ring is inserted in an exactly matching-shaped tunnel (i.e. a circle), only mass effect will be present. However, as previously demonstrated, the tunnel in ectatic corneas forms in the shape of two opposite halves of two ellipses. Therefore, the interaction between the circular ring and the elliptical tunnel will create an intrinsic force that would alter the corneal curvature in addition to the ring’s “mass effect”. While some authors might consider the discrepancy between the ring curve and the channel curve as a complication of surgery \cite{18}, we think that better understanding of this phenomenon will help predict the right position of the ring and its effect. If the channel is steeper then the ring, centrifugal “hoop stress” will be generated to stretch the cornea toward the periphery and more flattening will result. If the ring is steeper than the channel, centripetal “hoop stress” forces will be generated which may counteract the “mass effect” of the ring and

Figure 9 Corneal topography of a 14 years old patient diagnosed with keratoconus presenting high refractive errors and low BCVA. The topography shows a highly irregular astigmatism; Two asymmetrical segments were inserted; (INTACS) 150 degrees -450 μm inferiorly and 150 degrees-250 μm superiorly. Incision axis was placed at 10 degrees.

Figure 10 Large pellucid like advance ectasia is treated with large arch length 210 degrees ICRS. Incision site is chosen in order to place the ring under the steep zone of the cornea in order to embrace the irregularities. In this case, incision axis is placed at 50 degrees and the 2 tips of the 210 degrees ring segment are placed at the 35 degrees and 185 degrees.

Topography guided nomogram for intracorneal ring segment implantation
ring’s effect might be reversed toward steepening more than flattening. This could explain in part the vision improvement experienced after the removal of the superior ring in certain patients with inferiorly displaced cones\textsuperscript{[19]}. The net result is that the same ring will have a more pronounced flattening effect in the steep area of the cornea than in the flat area and thick segments should be implanted under the steep portion of the cornea regardless of the cylinder axis of refraction (which is not constant in case of keratoconus). The one vs two rings’ decision is made according to the location of the steep cornea (above and below the centre), and according to the amount of spherical equivalent to be corrected. One ring insertion is practiced in order to reduce irregular astigmatism and enhance the BCVA. In case of keratoconus with high refractive errors and poor BCVA, one ring segment is inserted before the phakic intraocular lens implantation in order to reduce irregularities and enhance BCVA (Figure 11). Low to moderate refractive errors, 1 or 2 asymmetric segments are inserted in order to reduce irregularities and enhance uncorrected distance visual acuity (Figure 12) (Dr Jarade’s nomogram for the treatment of keratoconus\textsuperscript{[20]}). Asymmetric irregularities would require asymmetric rings (different in thickness and arch length if necessary). In general, one ring under the steep part of the cornea will produce the desired regularization that would result in BCVA improvement. However, if a decrease in spherical equivalent is desirable, then two asymmetric rings are indicated. According to our surgical experience, the new nomogram provided more topographic regularity with significant reduction of astigmatism and better improvement in UCVA and BCVA than the conventional nomogram. This new concept might be integrated into the femtosecond laser software to create topography-guided, customized, elliptical tunnels with modified focal asphericity that allows for customized focal flattening of the irregularly steepened ectatic cornea. Also, a comparative study-in similar ectatic corneal cases-comparing
the results of our new nomogram to the conventional ones is needed in order to validate our results and help better understanding of the corneal response to ICRS insertion in keratoconus

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